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VIBRATION CHARACTERISTICS OF A QUARTER CAR MODEL MOVING ON ROAD SURFACE WITH RANDOM ROUGHNESS

Abstract

This article explores vibration characteristics of quarter car vehicle model moving on road surface with random roughness using Monte - Carlo (MC) simulation method. The roughness of road surface is modeled as a non-stationary stochastic process in spatial domain. The governing equation of vehicle motion under the condition of road surface takes a form of stochastic differential equation system and is solved by the MC method with Ito's formulation. Vibration response of the vehicle is analyzed for different cases of road surface quality.

Key words:

quarter car, random roughness, Monte-Carlo simulation, random response;

1. Introduction

The vibration of vehicle systems becomes an object of research interest from manufacturers and scholars because the vehicle vibration appears as a factor that affects directly the comfortableness of the occupants experiencing on the vehicle, the durability, service life and many other technical aspects [1]. In framework of the present study, we are concerned with the suspension system of vehicles with a model of two-degree-of-freedom system. The analysis on the dynamics and control of the vehicle's suspension system when the vehicle is moving on the road surface is an important task. For simple suspension systems using the quarter car model, modern control methods such as fuzzy control, adaptive control, etc. [3] can be used to achieve the goal of occupant comfort. In fact, when the vehicle is moving on the road, the road surface may have different roughness that greatly affects the vehicle's response during motion. It would be practical if the road surface was treated as a stochastic process instead of considering as a conventional plane [4]. The statistical characteristics of the road surface will affect the movement of vehicles through the contact of the tires with the road surface and the speed of the vehicle. In [5], Agharkakli et al. analyzed and simulated active and passive suspensions for a vehicle model with different road surface data. Stutz and Rochinha [6] have proposed a control method for a quarter vehicle model subjected to random pavement excitation. These authors have used a model with two-degrees-of-freedom to obtain vehicle response in both active and passive control scenarios. In a recent study of vehicle suspension systems [7], the Kalman-type filter has been proposed based on different pavement classifications, in order to evaluate the impact of pavement on the quality of handling and responsiveness of the vehicle.

In this article, using the Monte-Carlo simulation method, the authors analyze vibrations of quarter car model moving on a random road surface, in which the wheel is modeled as a spring-mass system, and the vehicle is modeled as a damper-spring-mass system attached to the spring-mass system of the wheel. The entire vehicle model is a two-degrees-of-freedom system moving on a random surface. Parameters of random surface are taken according to *Standard ISO/TC108/WG9* [8,9].

2. Characteristics of road surface roughness

2.1. Random roughness model

Let $q(s)$ be the function describing the road surface roughness (RSR) at position *s*. The RSR is considered as a stationary stochastic process in a spatial domain similar to that in the time domain. If a vehicle is traveling at a constant speed, taking into account the relationship between the traveling distance and time, the RSR can be considered to be a stationary stochastic process in the time domain (since *distance = velocity* $\times$ *time*).

It has been found that when the vehicle is moving with a variable velocity, the tire response is essentially a non-stationary stochastic process in the time domain.

The power spectral density (PSD) of the road surface roughness in the spatial domain can be expressed as follows:

$$S_q(n) = S_q(n_0)\frac{n_0^2}{n^2} \quad (1)$$

where $S_q(n_0)$ is the roughness coefficient; the quantity n_0 is the reference spatial frequency, $n_0 = 1/(2\pi) \approx 0.1592$; $S_q(n)$ is the spatial power spectral density -dependent *spatial frequency* n (unit $[n] = 1/\mathrm{m}$). The relationship between spatial angular frequency and spatial frequency n is given by:

$$\Omega = 2\pi n \quad (2)$$

The above relationship is similar to that between angular frequency and time frequency in mechanical systems. Withdrawing n from (2) and substituting on the right side of (1), we get an expression of the energy spectral density function corresponding to the spatial angular frequency Ω:

$$S_q(\Omega) = \frac{S_q(n_0)n_0^2}{\Omega^2} \quad (3)$$

We find that if $\Omega \to 0$ then $S_q(\Omega) \to +\infty$. This is physically unreasonable. Therefore, a modified form of the energy spectral density (3) is introduced as follows:

$$S_q(\Omega) = \frac{S_q(n_0)n_0^2}{\Omega^2 + \Omega_c^2} \quad (4)$$

where $\Omega_c = 2\pi n_c$ is *the cut-off frequency* [*the frequency at which energy flowing through the system begins to be attenuated rather than passing through*]. With the new form (4), if $\Omega \to 0$ then $S_q(\Omega) \to S_q(n_0)n_0^2/\Omega_c^2$ has a finite value. On domain $\Omega \geq 0$, it is clear that $S_q(\Omega)$ is a function that decreases with increasing Ω. When Ω is quite large, $S_q(\Omega)$ becomes quite small. Because there is a cut-off frequency Ω_c, we only consider the frequencies in the domain $0 \leq \Omega \leq \Omega_c$.

Equation (4) can be considered as a resulting response of a first-order linear differential equation system with input white noise process W:

$$\frac{dq(s)}{ds}+\Omega_c q(s)=n_0\sqrt{S_q(n_0)}\,W(s) \tag{5}$$

where s is a spatial variable; $W(s)$ is the stationary white noise process. It is seen that $q(s)$ is a stationary random response in the spatial domain.

The corresponding transfer function $H(\Omega)$ of system (5) takes the form:

$$H(\Omega)=\frac{n_0\sqrt{S_q(n_0)}}{\Omega_c+i\Omega} \tag{6}$$

where $i=\sqrt{-1}$ is an imaginary unit.

Notice that $ds=v(t)dt$. Then we have the derivative representation in spatial variable s versus the derivative with respect to the time variable t:

$$\frac{dq}{ds}=\frac{1}{v}\frac{dq}{dt} \tag{7}$$

where $q=q(t)$ is a function of time. The function $q(t)$ can be understood as the response of the car's wheel on the road and depends on the velocity of the vehicle. Whether the vehicle goes fast or slow affects the instantaneous value of the response q Substituting (7) into (5), we get the equation of the time variable t:

$$\frac{dq}{dt}+v\Omega_c q(t)=n_0 v\sqrt{S_q(n_0)}\,W(s(t)) \tag{8}$$

Here, since two functions $v=v(t)$ and $s=s(t)$ depend on time t, $W(t)=W(s(t))$ is a non-stationary white noise process. We need to convert this non-stationary white noise process into a stationary white noise process. The technique of equivalence covariance is one of the effective tools to perform the above transformation. Its idea is to introduce a stationary white noise process whose covariance is equal to the covariance of the non-stationary white noise process. Accordingly, we introduce a stationary white noise process, $W_1(t)$, instead of a non-stationary white noise process, $W(s(t))$.

We only consider the case where the velocity $v > 0$ of the vehicle is constant. Suppose that the white noise process $W(s(t))$ satisfies the following correlation property:

$$\begin{aligned} &E\left[W\left(s(t_1)\right)W\left(s(t_2)\right)\right] = \delta\left(s(t_2) - s(t_1)\right) \\ &= \delta\left(v(t_2 - t_1)\right) = \frac{1}{|v|}\delta(t_2 - t_1) = \frac{1}{v}\delta(t_2 - t_1) \end{aligned} \tag{9}$$

where we used the Dirac-delta function equality, $\delta(vt) = \left(1/|v|\right)\delta(t)$.

Assume that a stationary white noise process $W_1(t)$, satisfies the following property:

$$E\left[\frac{W_1(t_1)}{\sqrt{v}}\frac{W_1(t_2)}{\sqrt{v}}\right] = \frac{1}{v}E\left[W_1(t_1)W_1(t_2)\right] = \frac{1}{v}\delta(t_2 - t_1) \tag{10}$$

Comparing (9) and (10), we can see that the covariances of $W(s(t))$ and $W_1(t)/v$ are equivalent. Equation (8) becomes:

$$\frac{dq}{dt} + v\Omega_c q(t) = n_0\sqrt{S_q(n_0)v}\,W_1(t) \tag{11}$$

where Ω_c is the cut-off frequency of the energy spectral density function, v is constant velocity.

2.2 Numerical simulation for road surface roughness

Figure 4 shows level values of road surface roughness through the response of equation (11) in a statistical sample with roughness coefficient $S_q(n_0) = 64\times10^{-6}$ (m^3) while the vehicle is traveling at velocity 5 ($\mathrm{m/s}$). Roughness coefficient 64×10^{-6} (m^3) is considered to be an average level according to *ISO/TC108/WG9 Standard* as shown in Table 1. The velocity 5 ($\mathrm{m/s}$) (~18 $\mathrm{km/h}$) is a slow velocity for a car. The peak value of the response, $q(t)$, is about 2.975$\times10^{-3}$ (m), which is a rather small value. The vehicle does not seem to be "bumpy" according to the driver's perception.

Table 1

Classification of road surfaces based on the value of the function, $S_q(n_0)$, according to ISO/TC108/WG9 Standard, 1972

Status description	$S_q(n_0)$ [x10^{-6} m^3]
Good	16 $(=2^4)$
Average	64 $(=2^6)$
Bad	256 $(=2^8)$
Very bad	1024 $(=2^{10})$

Figure 5 is the evolution of road surface roughness or asperity when the vehicle is still traveling at a slow velocity 5 ($\mathrm{m/s}$) but the road surface roughness coefficient is $S_q(n_0)=1024\times10^{-6}$ (m^3), which means the road is "bad" according to the classification in Table 1. Since the road is quite bad, the response *q(t)* will increase more than that of 64×10^{-6} (m^3) as shown in Figure 4. The peak value is around 10.304$\times10^{-3}$ (m), much higher than the average case. At this time, the vehicle goes quite "bumpy" according to the driver's perception.

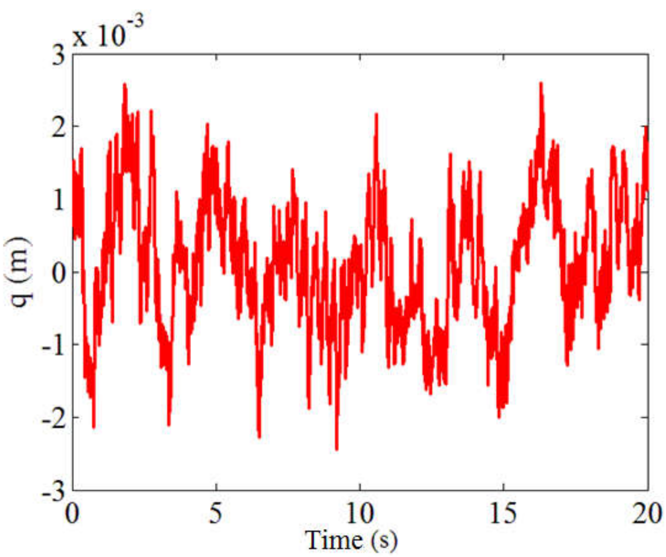

Figure 4 – Road surface response with roughness parameter $S_q(n_0)=64\times10^{-6}(\mathrm{m}^3)$ with vehicle velocity $v=5\,(\mathrm{m/s})$

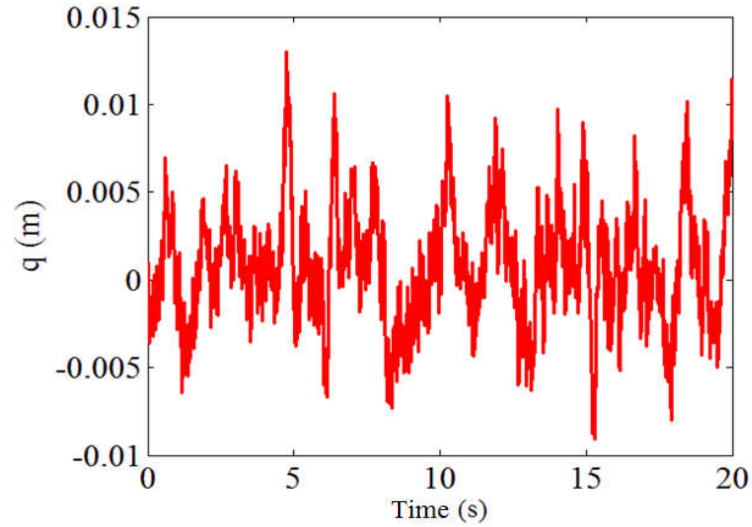

Figure 5 – Road surface response with roughness parameter
$S_q(n_0) = 1024 \times 10^{-6}$ (m^3) with vehicle velocity $v = 5$ ($\mathrm{m/s}$)

Table 2

Statistics of road surface data from random simulation

Velocity/Speed ($\mathrm{m/s}$)	**Roughness coefficient** (m^3)	**Mean squared response** (m^2) [in 1000 simulation samples]	**Peak value** (m) [in a random sample)]
5	64×10^{-6}	0.833×10^{-6}	2.975×10^{-3}
5	1024×10^{-6}	13.207×10^{-6}	10.304×10^{-3}
30	64×10^{-6}	0.955×10^{-6}	3.534×10^{-3}
30	1024×10^{-6}	15.351×10^{-6}	13.143×10^{-3}

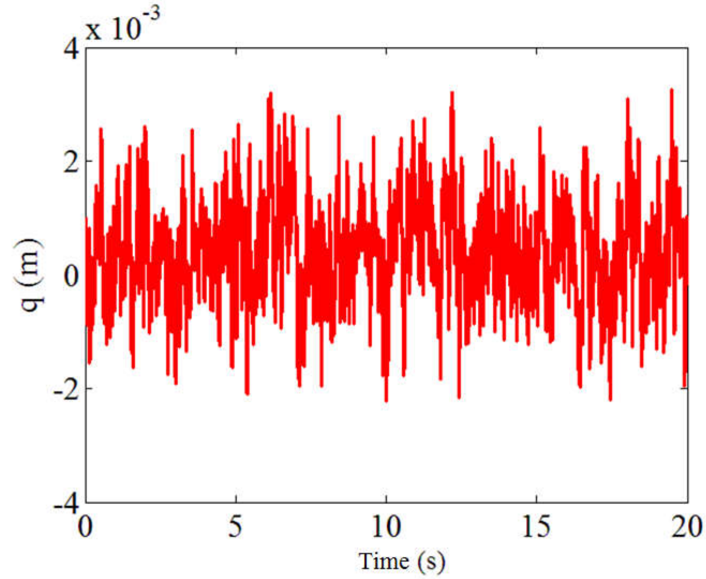

Figure 6 – Road surface response with roughness parameter

$S_q(n_0) = 64 \times 10^{-6}$ (m^3) with vehicle velocity $v = 30$ (m/s)

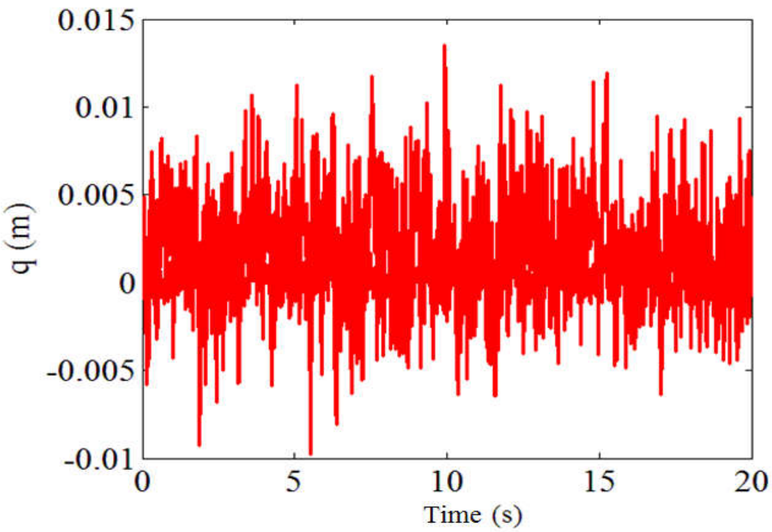

Figure 7 – Road surface response with roughness parameter

$S_q(n_0) = 1024 \times 10^{-6}$ (m^3) with vehicle velocity $v = 30$ (m/s)

Figure 6 shows that when the vehicle is traveling at a high velocity of 30 (m/s), on the average road surface, the response *q(t)* is quite small. The response peak value is about 3.534×10^{-3} (m) in a random sample.

In the case of the vehicle traveling at high velocity on a bad road, the response q(t) will also increase compared to that on an average road. It is illustrated in a statistical sample in Figure 7.

Table 2 describes some road surface data for different speed and roughness cases. If we perform a sufficiently large number of simulations, for example, the number of simulations is 1000 samples in Table 2, the simulation time step is 0.01 (s), we find that the mean squared response of *q(t)* also increases in the case of increasing the level of roughness coefficient. From equation (11), we can see that the increasing roughness will lead to an increase in the input white noise intensity, so the response *q(t)* will have a larger value. This explains why in the above Table and in figures, the worse the road, the higher the response *q(t)* will be (most noticeable is the larger response peak).

3. Response of the vehicle moving on the road surface

Figure 8 illustrates a quarter car model moving on the road surface with the level of surface roughness represented by the response function *q(t)* described in the above section.

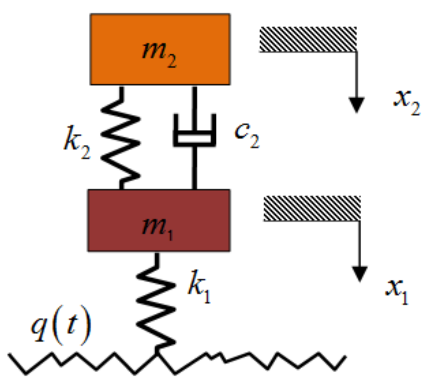

Figure 8 – Model of a vehicle system with two-degree-of-freedom moving on a random road surface

The wheel is modeled as an elastic spring of stiffness k_1 attached with mass m_1 in vertical direction. The body of a vehicle of mass m_2 placed on the wheel on a mechanism modeled as a spring of stiffness k_2. The body-wheel system is connected by an additional viscous damper with a damping coefficient c_2. The equation of motion of the vehicle under the condition of the road surface $q(t)$ is given by:

$$\begin{aligned} & m_1\ddot{x}_1 - c_2\left(\dot{x}_2 - \dot{x}_1\right) - k_2\left(x_2 - x_1\right) + k_1\left(x_1 - q\right) = 0 \\ & m_2\ddot{x}_2 + c_2\left(\dot{x}_2 - \dot{x}_1\right) + k_2\left(x_2 - x_1\right) = 0 \end{aligned} \tag{12}$$

Equation (12) can be written in the following form:

$$\begin{aligned} & \ddot{x}_1 - 2\mu\zeta_2\omega_2\left(\dot{x}_2 - \dot{x}_1\right) - \mu\omega_2^2\left(x_2 - x_1\right) + \omega_1^2 x_1 = \omega_1^2 q \\ & \ddot{x}_2 + 2\zeta_2\omega_2\left(\dot{x}_2 - \dot{x}_1\right) + \omega_2^2\left(x_2 - x_1\right) = 0 \end{aligned} \tag{13}$$

where

$$\omega_1 = \sqrt{\frac{k_1}{m_1}},\ \omega_2 = \sqrt{\frac{k_2}{m_2}},\ \zeta_2 = \frac{c_2}{2\sqrt{m_2 k_2}},\ \mu = \frac{m_2}{m_1} \tag{14}$$

The quantities ω_1, ω_2 are the fundamental frequencies of the system; ζ_2 is damping coefficient; μ is the ratio of the mass between objects m_2 and m_1. Since the system of equations (13) is linear, the system response can be easily obtained by an appropriate analytical or numerical method. Here, equation (12) is solved by Monte-Carlo simulation method. The system parameters are given in Table 3. To simplify the numerical solution, we rewrite the system (13) together with equation (11) into the following system:

$$\begin{aligned} & \dot{y}_1 = y_2 \\ & \dot{y}_2 = 2\mu\zeta_2\omega_2\left(y_4 - y_2\right) + \mu\omega_2^2\left(y_3 - y_1\right) - \omega_1^2\left(y_1 - y_5\right) \\ & \dot{y}_3 = y_4 \\ & \dot{y}_4 = -2\zeta_2\omega_2\left(y_4 - y_2\right) - \omega_2^2\left(y_3 - y_1\right) \\ & \dot{y}_5 = -v\Omega_c y_5 + n_0\sqrt{S_q\left(n_0\right)v}\,W_1\left(t\right) \end{aligned} \tag{15}$$

The Ito's formulation of equation (15) has the following form

$$\begin{aligned} & dy_1 = y_2 dt \\ & dy_2 = \left[2\mu\zeta_2\omega_2\left(y_4 - y_2\right) + \mu\omega_2^2\left(y_3 - y_1\right) - \omega_1^2\left(y_1 - y_5\right)\right]dt \\ & dy_3 = y_4 dt \\ & dy_4 = \left[-2\zeta_2\omega_2\left(y_4 - y_2\right) - \omega_2^2\left(y_3 - y_1\right)\right]dt \\ & dy_5 = -v\Omega_c y_5 dt + n_0\sqrt{S_q\left(n_0\right)v}\,dB_1\left(t\right) \end{aligned} \tag{16}$$

where $B_1(t)$ is the unit Wiener process.

The numerical results for the vehicle model on the road are illustrated in Figures 9-12. The vehicle velocity is taken as 5 (m/s). The system parameters appearing in equation (16) are calculated in Table 4 from the given original parameters in Table 3. The obtained damping coefficient $\zeta_2 = 0.2070$, is considered as weak damping. Body mass is 14.5838 times larger than wheel mass.

Table 3

Parameters of vehicle model

m_1 (kg)	m_2 (kg)	k_1 (N/m)	k_2 (N/m)	c_2 (Ns/m)
24	350	85270	9475	754

Table 4

Parameters of natural frequency, damping coefficient and mass ratio of the vehicle model

ω_1 (rad/s)	ω_2 (rad/s)	ζ_2	μ
59.6063	5.2030	0.2070	14.5833

In Figures 9 and 10, we see that when the vehicle is traveling on a medium quality road, the vehicle vibration is quite small. Since wheel vibrations are directly influenced by road surface, the response x_1 has a similar behavior to that of the pavement. However, the response x_2 behaves differently in terms of the period of oscillation; specifically, the period of oscillation of x_2 is larger than the period of oscillation of x_2 .

The vibration properties of the vehicle will become different, which is reflected in the larger vibration amplitude when the vehicle is traveling on a bad road with $S_q = 1024 \times 10^{-6}$ (m). These results are illustrated in Figures 11 and 12. This is very consistent with the real feeling when we go on bumpy roads. We also see that the response x_2 has a larger period of oscillation than the period of x_1.

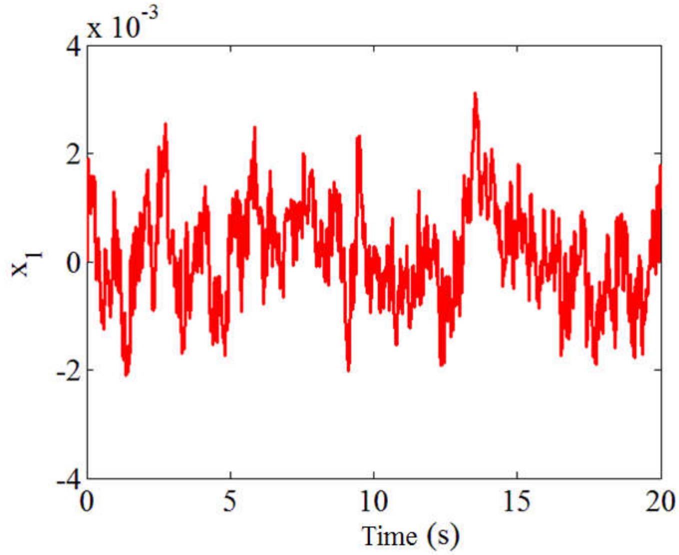

Figure 9 – Response x_1 when the vehicle travels at velocity

$v = 5$ (m/s) on a road with $S_q = 64 \times 10^{-6}$ (m)

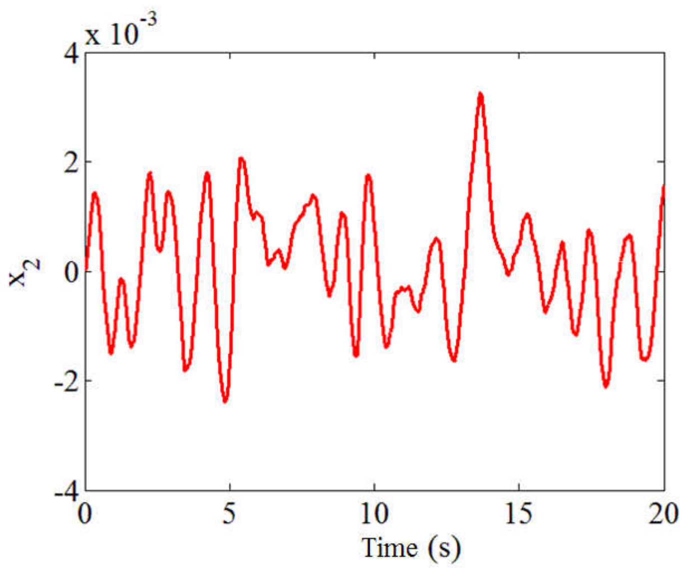

Figure 10 – Response x_2 when the vehicle travels at velocity

$v = 5$ (m/s) on a road with $S_q = 64 \times 10^{-6}$ (m)

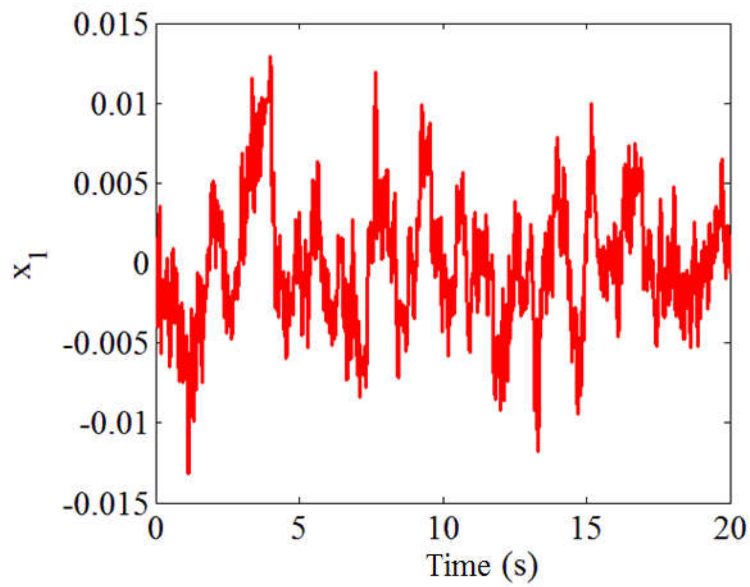

Figure 11 – Response x_1 when the vehicle travels at velocity

$v = 5$ (m/s) on a road with $S_q = 1024 \times 10^{-6}$ (m)

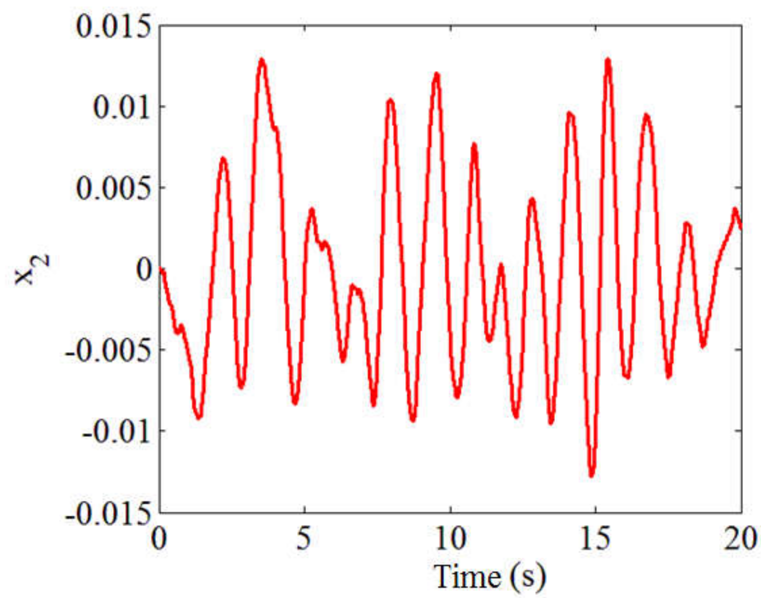

Figure 12 – Response x_2 when the vehicle travels at velocity

$v = 5$ (m/s) on a road with $S_q = 1024 \times 10^{-6}$ (m)

4. Conclusion

Ambient vibration is a source of energy harvesting that has many advantages such as availability, ease of exploitation and use. In this paper, the authors calculate and illustrate a case study of the quarter car response associated with randomness of road surfaces in which vibration of the system can be a potential source for vibration harvesting. The results obtained are as follows:

- Simulation of response of random road surfaces based on the spatial frequency spectrum with different roughness coefficients is caried out and can be used as data for calculating vibration of vehicles.

- An equation system of vehicle - road surface interaction has been established, then solved using the Monte-Carlo numerical simulation to obtain a random response of quarter car model.

- Response result of vehicle - road surface system with different road surface conditions is explored. This could be a certain suggestion for energy harvesting strategies from vehicle motion because this source of vibration is quite abundant and relatively easy to explore in our daily life.

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